

ME525 Applied Acoustics Lecture 9, Winter 2024

The acoustic point source (monopole)

The Green's function

Boundary conditions and acoustic doublet

Peter H. Dahl, University of Washington

Continuing with $ka \ll 1$ and $kL \ll 1$ limit, monopole source and acoustically compact source

In previous lecture we invoked the $ka \ll 1$ limit, and with minor rearrangement the pressure is

$$p(r, t) = -i\omega(\rho_0 u_0 4\pi a^2) \frac{e^{ikr}}{4\pi r} e^{-i\omega t} \quad (1)$$

Thus the strength of this acoustic source is defined by the time derivative of *mass flow*, or described another way, it is the *rate of change of mass flow* introduced per unit volume.

After this we defined an *effective source strength* by bundling everything and putting $q = -i\omega(\rho_0 u_0 4\pi a^2)$ giving,

$$p(r, t) = \frac{q}{4\pi r} e^{ikr - i\omega t} \quad (2)$$

where the source is at the center of the coordinate system and pressure is function only of radial coordinate r . So, this source no longer has any length scale a . This length scale has been removed on the assumption that $ka \ll 1$, and we can replace the sphere of radius a with effective source of strength q .

This defines the concept of concept of a point-like source or *acoustic monopole*. Such a source will generate wave motion in no preferred direction, producing a wave which spreads spherically outward. If there no boundaries, i.e., the medium of infinite extent, the waveform depend only on the range r from the center of the source, and not depend on the spherical angles α, ϕ as shown in Fig. 1 of Lecture 7.

Furthermore, such a source need not have originally in the form of an exact sphere. The source may instead have some complicated shape (Fig. 1) with characteristic length scale L . We arrive an extraordinarily useful rule: if the characteristic scale L of source is such that $L \ll \lambda$ where λ is the acoustic wavelength, then the source is *acoustically compact*, and can be viewed as a monopole source. Once the source is deemed acoustically compact the scale L is no longer relevant. The source can be modeling as Eq. (2), where the source strength, q is determined empirically by measurement.

For example, if p_{rms} is measured at range r m from the source, then we can estimate $|q|$ as follows

$$\frac{|q|}{4\pi} \frac{1}{\sqrt{2}} \frac{1}{r} = p_{rms} \quad (3)$$

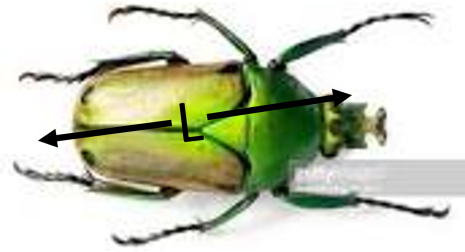


Figure 1: An acoustic source with characteristic scale L for which $kL \ll 1$ which can be modeled as an *acoustically compact* or source, or monopole.

giving at least a value for $|q|$. Often that is all we are after as the important physics relating to sound propagation is embodied in the factor $\frac{e^{ikr}}{r}$.

Continuing with the Green's function

The pressure at a *field point* \vec{r} , given a source at an arbitrary *source point* \vec{r}_0 that need not be at origin (Fig. 2) as follows:

$$p(\vec{r}, t) = \frac{q}{4\pi|\vec{r} - \vec{r}_0|} e^{ik|\vec{r} - \vec{r}_0| - i\omega t} \quad (4)$$

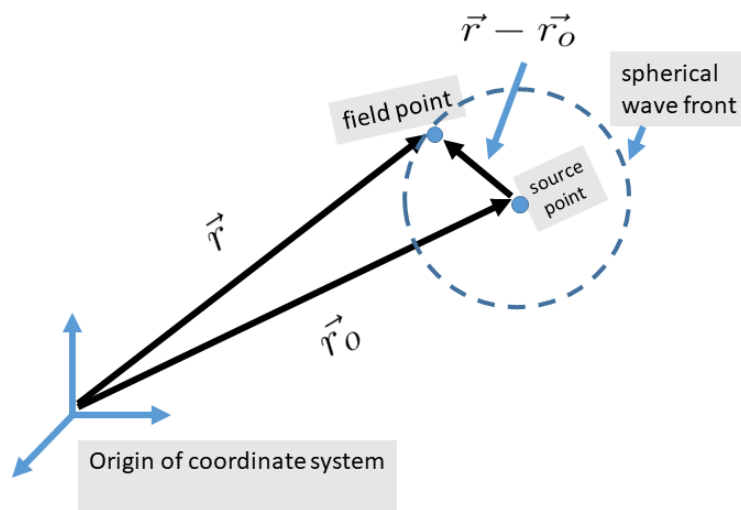


Figure 2: An acoustic source at the source point \vec{r}_0 producing the acoustic field at field point \vec{r} .

Equation (4) satisfies the inhomogeneous Helmholtz equation, for which the delta function on the RHS represents a point source of strength q at position \vec{r}_0 such that

$$(\nabla^2 + k^2)p = -q\delta(\vec{r} - \vec{r}_0) \quad (5)$$

Further compress notation by defining $R = |\vec{r} - \vec{r}_0|$ and remove the source constant q (until needed later), such that

$$g = \frac{e^{ikR}}{4\pi R}. \quad (6)$$

Call g the *free space Green's* that satisfies

$$(\nabla^2 + k^2)g = -\delta(\vec{r} - \vec{r}_0) \quad (7)$$

in an *unbounded* medium.

The Green's function is a model for sound propagation that is proportional to acoustic pressure, differing from pressure only by some multiplicative constant that can be complex. The constant may already be known from theory, or determined empirically by measurement. In many applications the constant isn't used as the Green's function usually embodies most if not all the important physics sound propagation.

Note the physical dimension of g is $1/L$. As currently constructed, g embodies all the range-dependent and phase properties of a sound field with point source located at \vec{r}_0 , but to bring a more useful dimension of pressure, g must be multiplied by a calibration constant. The particular form of g in Eq.(6) which concentrated or "impulse-like" in space is known as a *harmonic Green's function*. In this course we use primarily harmonic Green's function solutions, representing a single-frequency, or narrow band condition, and by Fourier superposition we can combine multiple frequencies.

The expression by way Eq.(5) is common in engineering and physics, where a field quantity, here sound pressure, is governed by a linear partial differential equation with an inhomogeneity at location \vec{r}_0 acting as a source term. This pressure field is everywhere smooth and analytic (possesses spatial derivatives that are not infinite), except at the source point where it can only be described by a delta function inhomogeneity. This is not unlike a wave created on water, for which the wave field is analytic—except at the point on that surface where the rock splashed (location of delta function) and produced this wave. The acoustic field generated by a delta function inhomogeneity is sometimes formally referred to as the *Green's function for the problem*.

Combination of two point sources to satisfy a boundary condition: the acoustic doublet

The Green's function of Eq.(6) is our most basic one which does not need to contend with boundary conditions such as an air-water interface. A simple but realistic boundary condition encoun-

tered in acoustics is an acoustic source below an air-water interface. This is called the Lloyd mirror problem and the boundary condition is called "pressure release" meaning the acoustic pressure field must vanish on the boundary. Such boundary, again when viewed from source below, is also referred to as an 'acoustically soft' surface.

The 'soft' description arises from the characteristic impedance of two acoustic media involved in the problem: water where ρc is of order $1.5 \cdot 10^6$, and air where ρc is ~ 400 . In general, sound impinging on a boundary backed by one medium with characteristic impedance much less than the medium supporting the incident field will also behave approximately as a pressure release boundary, or be 'acoustically soft'.

Note that opposite happens for airborne sound over water; in this case the upper medium is supporting the incident field, and with characteristic impedance of the upper air medium being much less than of the lower water medium the boundary can be considered 'acoustically hard'. For example, the hard walls of the underground garage (Lecture 1) behave approximately this way in view of small ρc of the air medium.

Later more general boundary conditions are addressed, as say between soft and hard tissue in medical ultrasound, or between sea water and the seabed, which requires a description of the material properties in terms of the characteristic impedance involved in each boundary material. For now, though, so long as an 'acoustically soft' (or 'acoustically hard') approximate description applies, some very simple combinations of Green's functions can be applied to satisfy the boundary condition.

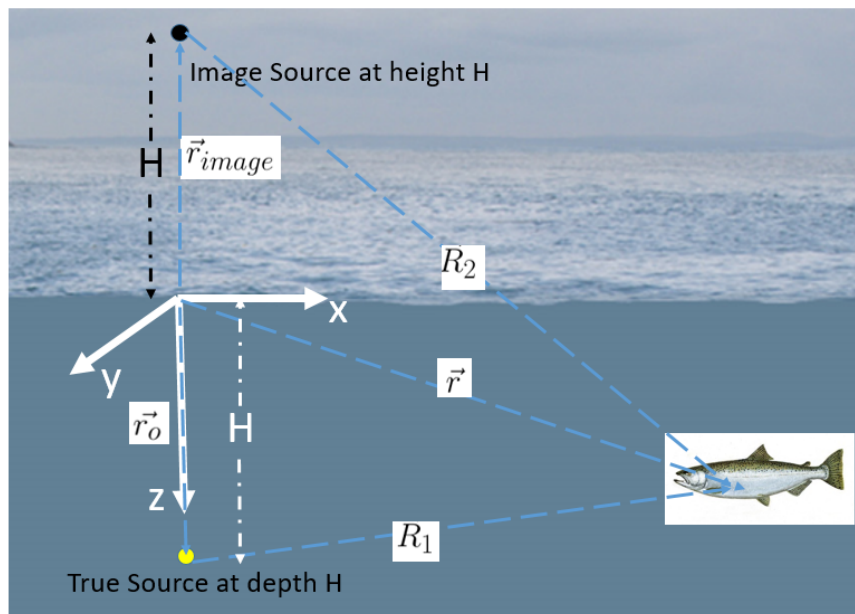


Figure 3: The geometry of the Lloyd Mirror problem showing a true source at depth H and image source at height H .

The pressure release boundary condition is satisfied by combining the true source with an image

source (Fig. 3). The image is of opposite sign such that combination of two sources along the boundary equals zero. In this case the Green's function take the following form

$$g = \frac{e^{ikR_1}}{4\pi R_1} - \frac{e^{ikR_2}}{4\pi R_2} \quad (8)$$

where $R_1 = |\vec{r} - \vec{r}_0|$ and $R_2 = |\vec{r} - \vec{r}_{image}|$. We can set $\vec{r}_0 = [0, 0, H]$ and $\vec{r}_{image} = [0, 0, -H]$ in terms of the x, y, z coordinates.

The combination of two free-space Green's functions as in Eq. (8) is known as *doublet*. You should convince yourself that this g equals 0 when evaluated at any x, y with $z = 0$, and remember that g is serving as a surrogate for pressure differing from it by some linear constant.

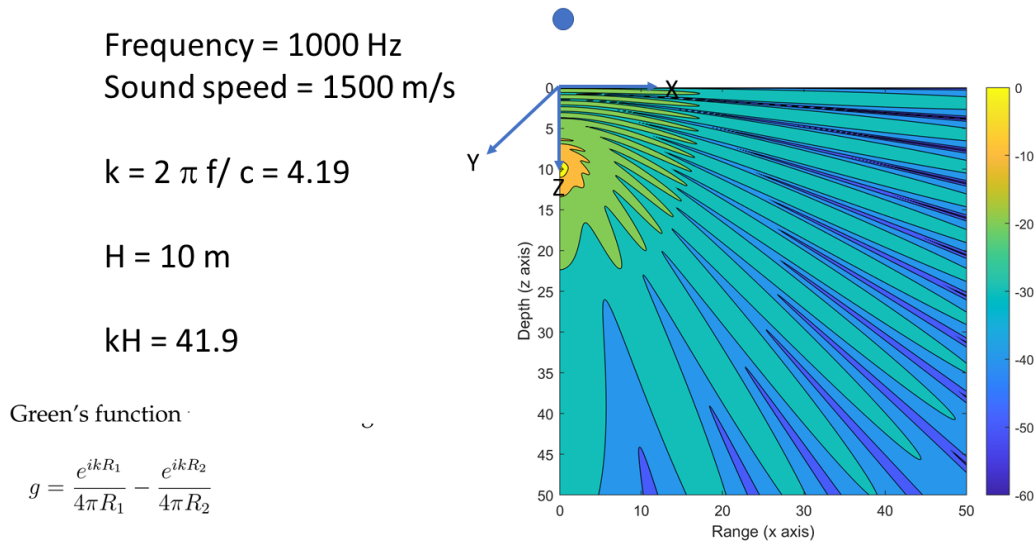


Figure 4: Field in dB with arbitrary reference for acoustic doublet for $H = 10$ and $kH = 41.9$

Three situations are discussed next for doublet based on differing depths H of the source (Fig. 4-6), and we'll see that a key parameter is kH . In such plots we are interested in how the "strength" of g or pressure, varies with x and z , and so $|g|$ (proportional to pressure amplitude) or $|g|^2$ (proportional to pressure-squared) is plotted. It thus makes sense to continue using decibels, and plot $10 \log_{10} |g|^2$. These three plots display azimuthal symmetry which makes it equally instructive to just plot $20 \log_{10} |g|$ in the x, z plane. Notice what is happening for when the parameter kH is getting smaller.

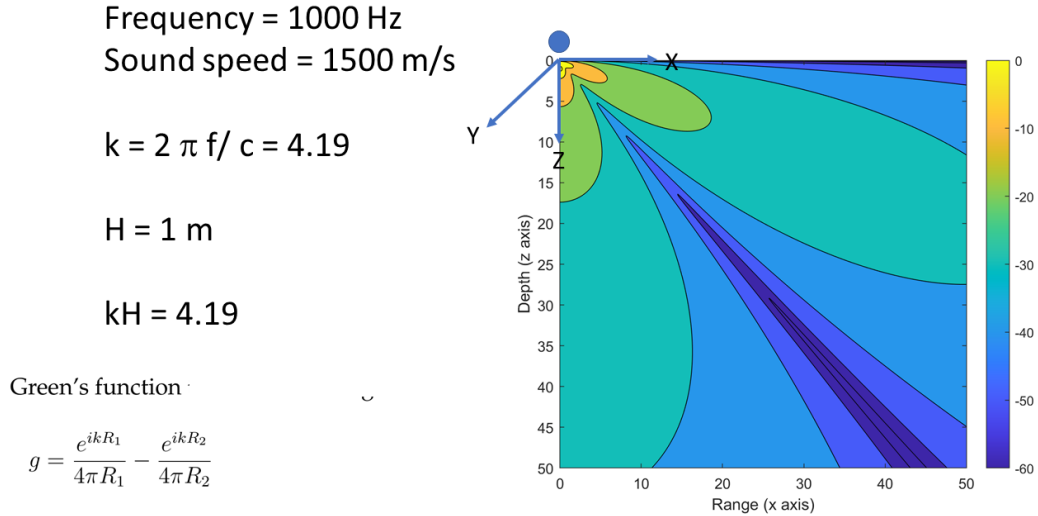


Figure 5: Field in dB with arbitrary reference for acoustic doublet for $H = 1$ and $kH = 4.19$

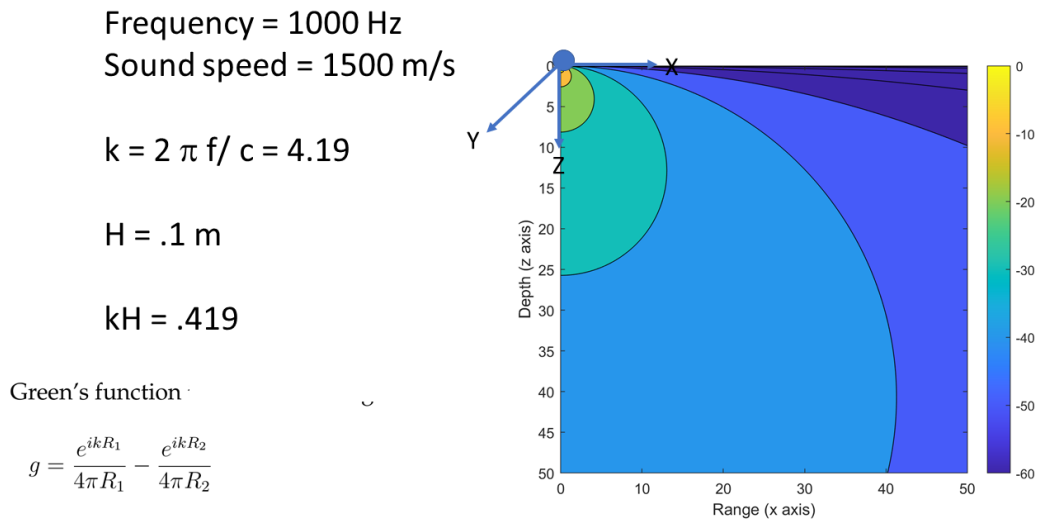


Figure 6: Field in dB with arbitrary reference for acoustic doublet for $H = 0.1$ and $kH = 0.419$

References

- Pierce, A. B, *Acoustics, An Introduction to its Physical Principals and Applications*, (Acoustical Society of America, and American Institute of Physics, 1989)
- Frisk, G. V. *Ocean and Seabed Acoustics* (Prentice Hall, Englewood Cliffs, NJ, 1994)
- L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics*, (John Wiley & Sons, New York, 1980)

ME525 Applied Acoustics Lecture 10, Winter 2024

the Green's function acoustic doublet and dipole

Peter H. Dahl, University of Washington

More on combination of two point sources to satisfy a boundary condition, and method of images

It pays to think about boundary conditions, e.g., between soft and hard tissue in medical ultrasound, or between sea water and the seabed, in terms of the characteristic impedance, $\rho_0 c$, of the acoustic medium on each side of the boundary. This does not tell the whole story, e.g., on one side or both sides there may be layers composed of differing sound speeds and densities, but it gives a good starting approximation.

But roughly speaking, for sound in a medium with high $\rho_0 c$ impinging on a boundary to another medium with very low $\rho_0 c$, the boundary condition will behave approximately as a 'pressure release' boundary, meaning the acoustic pressure must equal zero along this boundary. In the study of underwater acoustics, this pressure-release boundary conditions is assumed to be exact. To be perfectly exact assumes ratio: $\frac{(\rho c)_{air}}{(\rho c)_{water}}$ equals 0. Check it out using nominal values for each to confirm the ratio is quite small. Note that this ratio puts the characteristic impedance of medium from which sound originates (water) in the denominator.

Likewise if sound originates from air impinging water, the ratio is $\frac{(\rho c)_{water}}{(\rho c)_{air}}$ which is quite large. In this case, instead of the pressure going to 0, the pressure doubles at boundary. This is examined in more detail later.

The pressure release boundary condition is satisfied by combining the true source with an image source (Fig. 1). The image is of opposite sign such that combination of two sources along the boundary equals zero. In this case the Green's function for the Lloyd Mirror problem takes the following form

$$g = \frac{e^{ikR_1}}{4\pi R_1} - \frac{e^{ikR_2}}{4\pi R_2} \quad (1)$$

where $R_1 = |\vec{r} - \vec{r}_0|$ and $R_2 = |\vec{r} - \vec{r}_{image}|$. We can set $\vec{r}_0 = [0, 0, H]$ and $\vec{r}_{image} = [0, 0, -H]$ in terms of the x, y, z coordinates. The method used to find this Green's function is known as the *method of images* (Frisk).

Considering the symmetry of the acoustic field about the z axis (Fig. 2), we compute two cases with differing H as parameterized by $kH \gg 1$ and $kH < 1$ (Fig. 3). Adjust the parameter kH by changing either H or frequency which changes k . The plots are expressed in terms of contours of $10 \log_{10} |g|^2$, or $20 \log_{10} |g|$, in the x, z plane, a quantity we might convey to someone else (your co-

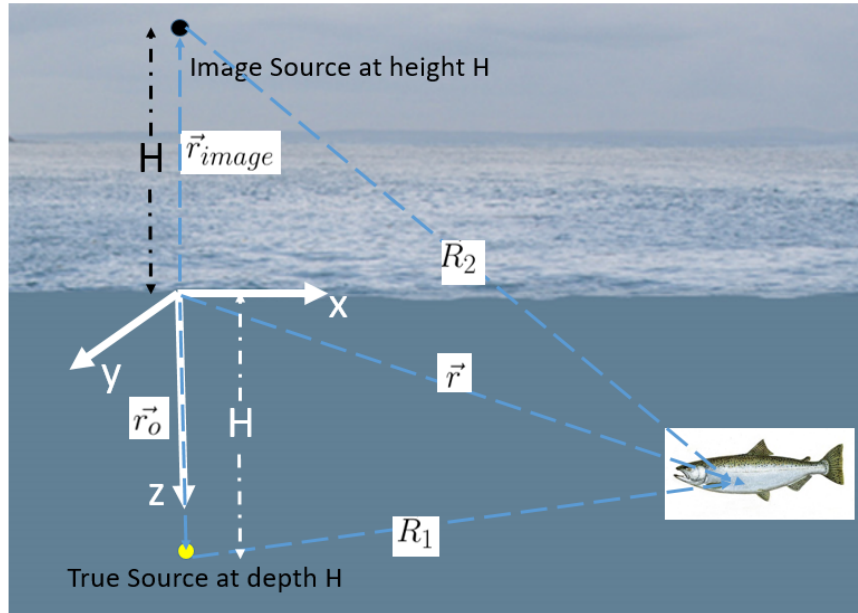


Figure 1: The geometry of the Lloyd Mirror problem showing a true source at depth H and image source at height H .

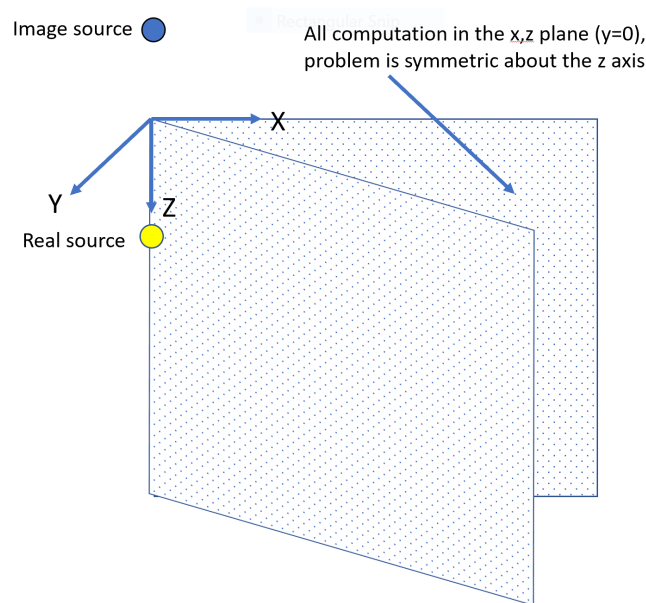


Figure 2: The acoustic field for the geometry in Fig. 1 is symmetric about the z -axis. Thus computing as function of x, z with $y = 0$ is completely sufficient.

worker, your research adviser, whomever) in terms of decibels (dB). But the dBs here are obviously not the same as, say, sound pressure level (SPL) in dB reference to $20\mu\text{Pa}$ (air) or $1\mu\text{Pa}$ (underwater). However, considering the fact that g is proportional to acoustic pressure, then there exists some constant-decibel offset to convert results in Fig. 3 to a SPL in dB reference to $1\mu\text{Pa}$.

Continuing with Fig. 3, notice the case $kH \gg 1$ shows about 14 lobes, or acoustic *beams*, for

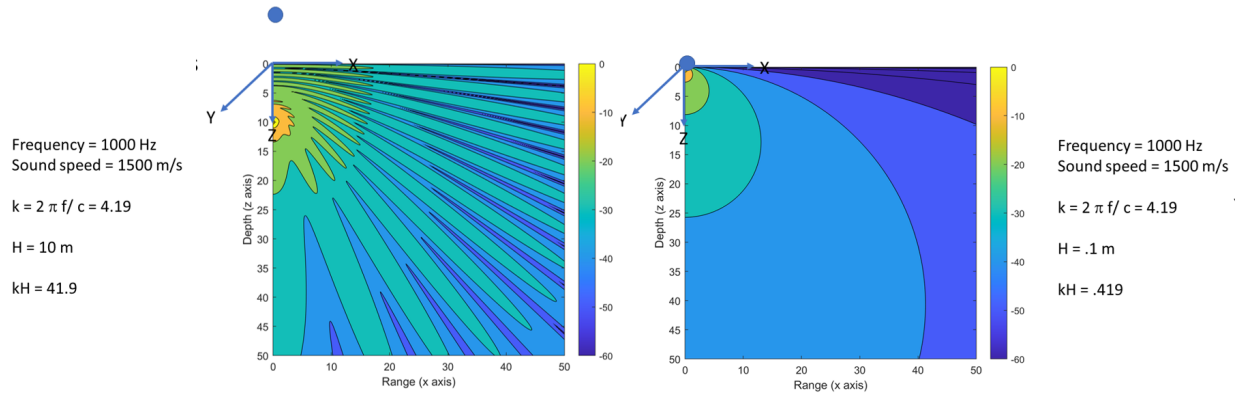


Figure 3: Field in dB with arbitrary reference for two cases with same frequency but differing H as parameterized by $kH \gg 1$ and $kH < 1$ acoustic doublet left: $H = 10$ m and $kH = 41.9$, right: $H = 0.1$ m and $kH = 0.419$. Similar plots in larger scale are shown in the power point as part of Lecture 9.

which $10 \log_{10} |g|^2$ varies between about -20 to -30 dB within a beam to about -60 dB outside the beam. A difference of 30 dB represents 1000-fold change in the value of $|g|^2$, hence the advantage in plotting as $10 \log_{10} |g|^2$. Regions where $|g|^2 \sim -60$ dB represent almost complete destructive interference, or *nulls* in the acoustic field. The interference is set up by superposition of contributions from the (positive) real source and (negative) image source. Though less clear for case $kH \gg 1$, there is very strong null along $z = 0$ boundary, where $|g| = 0$ (the decibel representation giving $-\infty$, a decibel level that is not captured in the contour plot.) In summary, with such a large variation in the field strength it pays to express it in terms of decibel level, and it is good practice to reserve the word 'level' when talking language involving decibels.

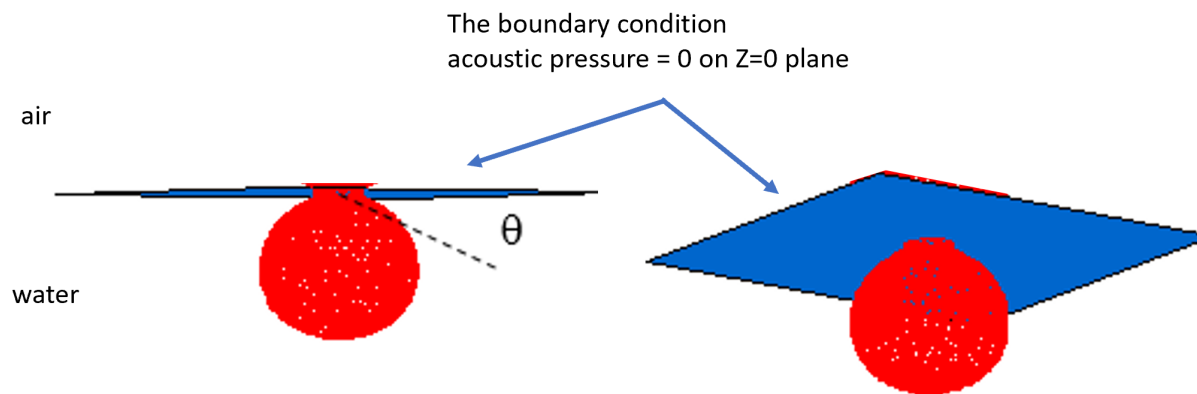


Figure 4: 3D rendition of the very broad beam which results for the acoustic doublet (Fig. 1) when $kH < 1$.

Turning now to the case $kH < 1$ in Fig. 3, there is also beam and null features, but now the lobes are quite large. For example, take the large -30 dB contour shaded area, and remember this contour swings around 360° about the z axis. It looks something like the rendition in Fig. 4, where

one angle with respect to the sea surface, θ , describes everything but dependence on range (an equivalent formulation involves an angle with respect to the z axis).

The acoustic dipole

Now look at the doublet for the case $kH \ll 1$ (i.e., not too different from Fig. 3 case $kH < 1$), where $2H$ is the separation between a source and its image which as opposite sign. Using the same coordinate system as in Fig. 1, recast R_1 as

$$R_1 = \sqrt{x^2 + y^2 + z^2 - 2zH + H^2} = r\sqrt{1 - 2zH/r^2 + H^2/r^2} \quad (2)$$

then evaluate

$$kR_1 = r\sqrt{k^2 - 2kzH/r^2 + (kH)^2/r^2} \quad (3)$$

In the limit $kH \ll 1$ the last term is ignored thus $R_1 \approx r(1 - zH/r^2)$, or put $R_1 = r - H \sin \theta$. Similarly we put $R_2 = r + H \sin \theta$. Now, the source \vec{r}_0 and image \vec{r}_{image} locations are out of the picture, everything is described by r and θ as in Fig. 5 (the depth coordinate z of the field point included in θ .)

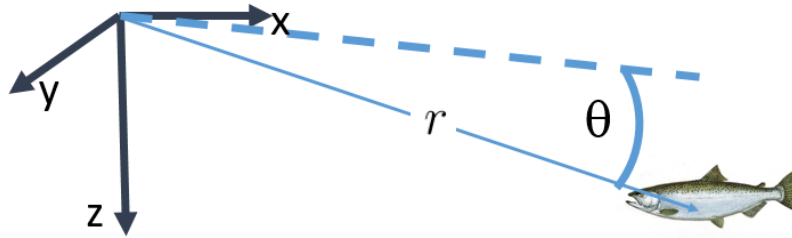


Figure 5: Situation describing a dipole located at center of coordinate system. The acoustic field is function of r and θ .

To recast this with Green's function that satisfies the free-surface boundary condition (discussed previously) composed of source and image of opposite sign (or doublet), again start with doublet form

$$g = \frac{e^{ikR_1}}{4\pi R_1} - \frac{e^{ikR_2}}{4\pi R_2} \quad (4)$$

with R_1 for the source and R_2 for the image as shown earlier. Analyze the doublet for the case $kH \ll 1$, where $2H$ is the separation between a source and its image.

The approximations for R_1 and R_2 , can now be inserted into Eq.(4). In doing so, encounter $e^{\pm ikH \sin \theta}$, which can be approximated as $1 \pm ikH \sin \theta$, consistent with the original $kH \ll 1$ as-

sumption. This leads to the final result for the Green's function, now referred to as a *dipole*

$$g = 2H \sin \theta \frac{e^{ikr}}{4\pi r} \left(-ik + \frac{1}{r}\right) \quad (5)$$

Observe once again the dimension of this Green's function is $1/L$, as is the case for the two monopoles (source and image) used for the Green's function in Eq.(1). However the dipole now has two terms—one which we will shown to fade away as $kr \gg 1$.

References

Frisk, G. V. *Ocean and Seabed Acoustics* (Prentice Hall, Englewood Cliffs, NJ, 1994)

ME525 Applied Acoustics Lecture 11, Winter 2024

Acoustic Dipole

Peter H. Dahl, University of Washington

The acoustic dipole strength $|f_D|$

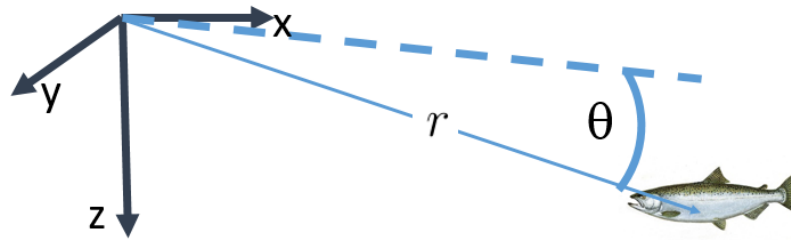


Figure 1: Situation describing a dipole located at center of coordinate system. The acoustic field is function of r and θ .

Let's continue our discussion on the basic dipole Green's function derived in the last lecture.

$$g = 2H \sin \theta \frac{e^{ikr}}{4\pi r} \left(-ik + \frac{1}{r}\right). \quad (1)$$

Recall Eq.(1) originates from the "doublet" consisting of two free-space Green's function sources of opposite sign

$$g = \frac{e^{ikR_1}}{4\pi R_1} - \frac{e^{ikR_2}}{4\pi R_2} \quad (2)$$

where these sources were separated by H and brought closer together such that $kH \ll 1$. Next incorporate a new "source strength" q to obtain pressure

$$p(r, t) = (q2H) \sin \theta \frac{e^{ikr}}{4\pi r} \left(-ik + \frac{1}{r}\right) \quad (3)$$

where harmonic time dependence $e^{-i\omega t}$ is assumed.

Apply now the same approach used to establish q for point source monopole, instead here allow $2H$ to shrink to 0 while increasing q to keep $(q2H)$ finite. Identify $|f_D|$ as this new quantity to replace $(q2H)$ which is called the *dipole strength*. Thus, we now more formally express the pressure from the dipole as

$$p(r, t) = |f_D| \sin \theta \frac{e^{ikr}}{4\pi r} \left(-ik + \frac{1}{r}\right). \quad (4)$$

Note the dimension of $|f_D|$: in MKS it must be N/m, or equivalently Pa-m and the dipole unlike

the monopole has two terms: one that dominates in the near field, $1/r$, and that dominates the far field ik .

Vector properties of the dipole

The dipole example just presented is one with the axis oriented perpendicular to the boundary (this axis being the line connecting the real source and image source, or z-axis as in Fig. 1). This is a very appropriate model, for example, for low frequency ship noise where $kH \ll 1$ with H being the depth of the noise generating mechanism such as the ship propeller, underwater noise caused by rain drops, and by bubbles bursting near the sea surface due to the action of wave breaking.

This dipole example also demonstrates how the boundary between air and water is satisfied upon either placing a dipole on that boundary, or in the case of the doublet, placing a source a distance H below the boundary and a negative image a distance H above the boundary. But more generally we want to place a dipole anywhere in space to represent a source with properties of two closely-spaced monopole sources of opposite phase (or sign), i.e. there is no requirement to have the dipole source be associated with a boundary. The dipole can be oriented in any direction and this direction will be the *dipole axis*.

To do this, make the strength $|f_D|$ represent the magnitude of a vector \vec{f}_D called the *dipole moment vector*, where \vec{f}_D is aligned with the dipole axis, and by convention points towards the positive side of the dipole. Thus \vec{f}_D points downward in Fig. 1, or in the case just described where the dipole represents a source very close (in the sense of $kH \ll 1$) to the air-water interface. To use \vec{f}_D in a more general orientation we need to restore a full vector description \vec{r} for the field point, with the angle α (Fig. 2) given by

$$\cos \alpha = \frac{\vec{f}_D \cdot \vec{r}}{|\vec{f}_D| |\vec{r}|} \quad (5)$$

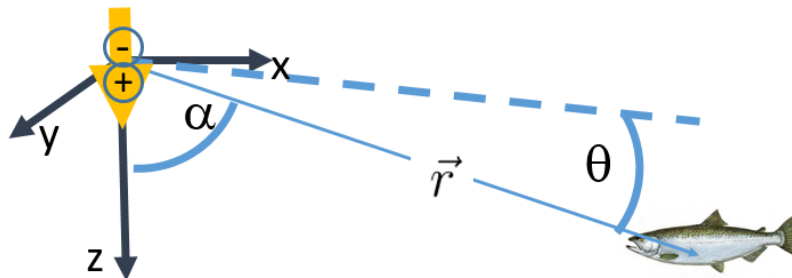


Figure 2: Showing orientation of the dipole moment vector (orange arrow), with positive source \oplus underwater and negative source \ominus above. The field point to the fish is now described with vector \vec{r}

The dipole moment vector in arbitrary orientation is shown in Fig. 3, and the pressure at the field point (with time dependence $e^{-i\omega t}$) is given by

$$p(FP) = \frac{1}{4\pi} \frac{\vec{f}_D \cdot (\vec{r} - \vec{r}_0)}{|\vec{r} - \vec{r}_0|^3} e^{ik|\vec{r} - \vec{r}_0|} \left(-ik + \frac{1}{|\vec{r} - \vec{r}_0|} \right) \quad (6)$$

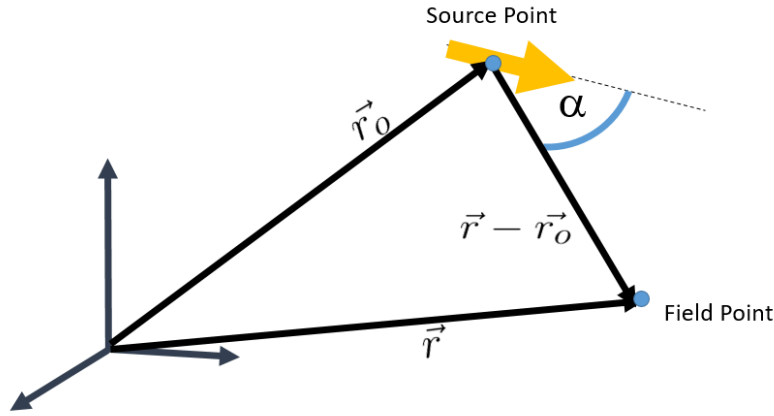


Figure 3: Showing orientation of the dipole moment vector (orange arrow), the dipole source location \vec{r}_0 and field point \vec{r} with arbitrary orientation within a coordinate system

Return now to the free space Green's function with harmonic time dependence

$$g = \frac{1}{4\pi} \frac{e^{ik|\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|} \quad (7)$$

where a pressure at some field point (FP) is

$$p(FP) = qg \quad (8)$$

Note that gradient of g is ∇g is given by (e.g, see Pierce)

$$\nabla g = \frac{1}{4\pi} \frac{(\vec{r} - \vec{r}_0)}{r} \left(ik - \frac{1}{r} \right) \frac{e^{ikr}}{r} \quad (9)$$

where $r = |\vec{r} - \vec{r}_0|$. Thus evidently Eq. (6) can also be written in the highly compacted form

$$p(FP) = -\vec{f}_D \cdot \nabla g \quad (10)$$

with Eq.(7) and Eq.(9) now representing our two fundamental source types, with monopole related to g and dipole related to ∇g .

Example: Dipole source on tower

Once again find Roy walking on snow along a path shown below. The boundary situation is that

heavy snow absorbs all sound reaching ground level. The tower at height 10 m above the snowy surface has two loudspeakers, placed together. The speakers are small of length L about 10 cm, and separated by 1 cm. The speakers are each generating a 59 Hz tone, but in “anti-phase”, i.e., the voltage supplied to one of the speakers is inverted (-) compared to the other (+) speaker. This results in a dipole source with dipole axis aligned with y axis shown in the figure. (Direction of dipole moment vector orientated towards negative y direction, consistent with convention of pointing towards the positive speaker source.) The amplitude of this 2-speaker dipole is such that the absolute value of the dipole moment vector $|\vec{f}_D|$ equals 100. This information, along with dipole axis aligned with y axis is sufficient to establish \vec{f}_D as follows:

$$\vec{f}_D = 100 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Take care to establish \vec{f}_D , think of magnitude, 100, and direction towards south or along negative y .

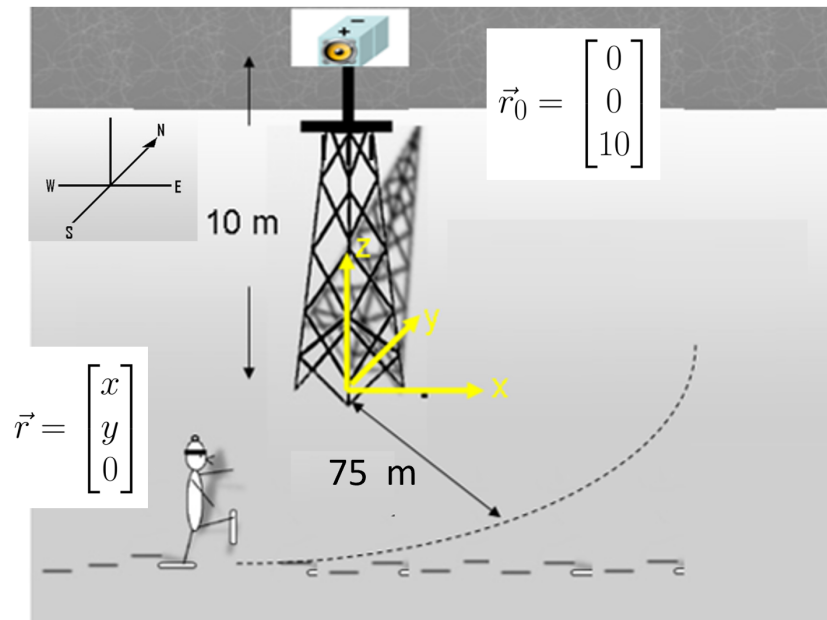


Figure 4: Roy walks around a dipole source in heavy snow which absorbs all sound that reaches ground level.

All ingredients are in hand to evaluate Eq.(6), which can be done in Matlab. Four example points based on changing the field point \vec{r} (Fig. 5) represent Roy starting at due south of the tower, southeast, exact east, and finally due north of the tower. Notice that the field vanishes when Roy is 90° or east of the tower. Unlike the monopole, the dipole source shows directivity, or angular variation of the acoustic field.

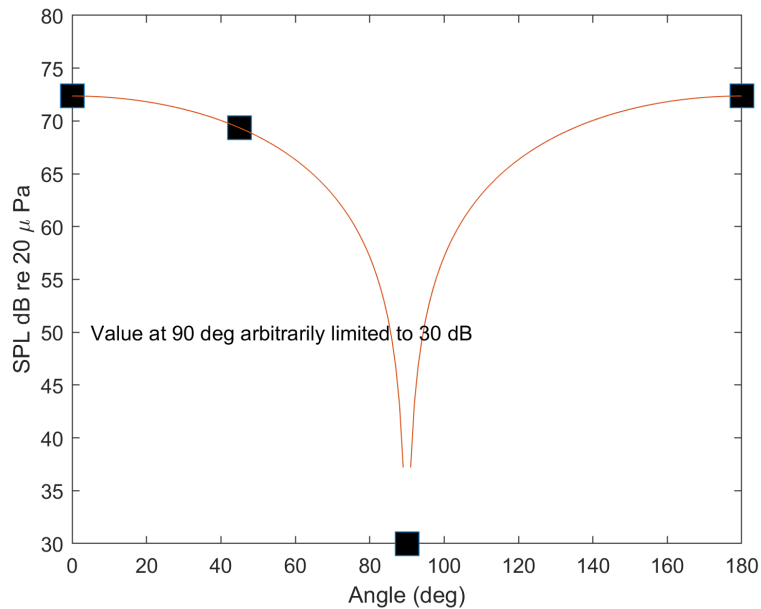


Figure 5: Sound pressure level (SPL) that Roy experiences in walking around the tower at constant radius 75 m, 180° of this walk is shown starting at $y = -75$, $x = 0$ (south of tower). Notice that the field vanishes when Roy reaches 90° (east of tower). Four points are shown as worked out in class.

Directivity

Dan Russell and colleagues performed a simple, illustrative experiment to demonstrate the concept of directivity from monopoles, dipoles and quadrupoles (a combination of two dipoles). His experiment (Fig. 6) is as follows: four speakers are on turntable and continuous (harmonic or single frequency) sound is broadcast at fixed frequency = 250 Hz. The RMS pressure is measured by a sound level meter (SLM) as the turntable rotates. Speakers (the square boxes) are arranged to be spatially packed together, separated by distance H , say.

The frequency and H are such that $kH \ll 1$ for this experimental configuration. Therefore, four speakers when broadcasting with the same phase (the black dots) can be considered a single monopole—see (a) top of Fig. 7. Next, take two of the speakers and reverse the wires (polarity) such that the phase is "negative", as in (b) top of Fig. 5. This can be considered a single dipole.

The lower part of Fig. 7 shows some results from the Russell *et al.* study. Directivity for the monopole is what we intuitively expect: regardless of the position of the turntable, the SLM gives the same result, and this result, rms pressure plotted in terms of dB, forms a circle. In contrast, the dipole directivity exhibits a deep null in the acoustic response as the turntable passes through 90 and 270°. The two halves of the dipole (in this case the two positive and two negative speakers) exhibit an exact phase cancellation and the acoustic pressure resulting from the coherent sum of four sources should vanish. The Russell *et al.* work also measures the acoustic field from a quadrupole

(see (d) in upper part of Fig. 7). This will exhibit a kind of clover-leaf directivity pattern.

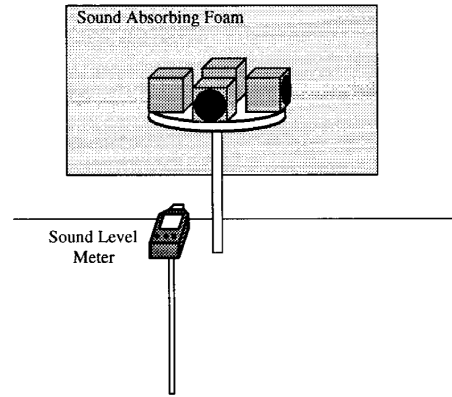


Figure 6: Apparatus for measuring directivity of monopoles, dipoles, and quadrupoles. This is Fig. 5 from Russell *et al.*, "Monopoles, Dipoles and Quadrupoles: An experiment revisited", Am. Journal of Physics, 1999.

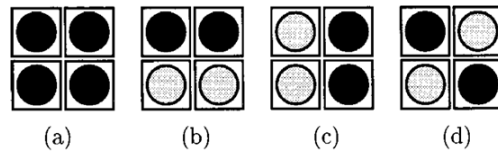


Fig. 8. Speaker arrangement and polarities for audible demonstration of sound power radiated by (a) monopole, (b) and (c) dipole, and (d) quadrupole sources.

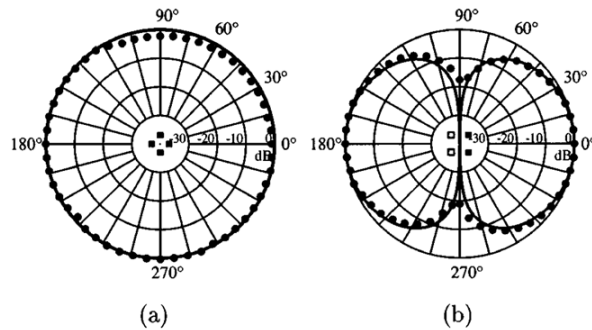


Figure 7: Measurements of directivity pattern for (a) monopole and (b) dipole. This is portion of Fig. 8 from Russell *et al.*, "Monopoles, Dipoles and Quadrupoles: An experiment revisited", Am. Journal of Physics, 1999.

Arrangement of 4 speakers, separated by H and driven by a frequency such that $kH \ll 1$ amounts to an idealized directivity problem where measurement well predicted by simple theory.

More often for realistic noise emission problems, an empirical measurement is required as in the case of directivity of jet noise measured on the ground (Fig. 8).

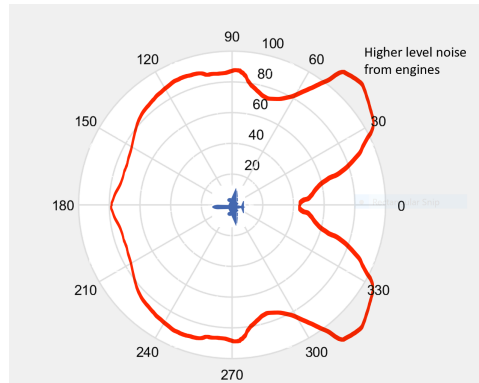


Figure 8: Notional directivity of jet noise emission over broad range of frequencies as function of angle.

References

Pierce, A. B, *Acoustics, An Introduction to its Physical Principals and Applications*, (Acoustical Society of America, and American Institute of Physics, 1989)